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Holographic dark energy and late cosmic acceleration

Diego Pavón

Departmento de Física, Universidad Autónoma de Barcelona, 08193 Bellaterra, Spain

E-mail: diego.pavon@uab.es

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Abstract
It has been persuasively argued that the number of effective degrees of freedom of a macroscopic system is proportional to its area rather than to its volume. This entails interesting consequences for cosmology. Here we present a model based on this 'holographic principle' that accounts for the present stage of accelerated expansion of the Universe and significantly alleviates the coincidence problem also for non-spatially flat cosmologies. Likewise, we comment on a recently proposed late transition to a fresh decelerated phase.

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1. Introduction

Nowadays there is an ample consensus, deeply rooted in observational grounds, that the Universe is currently undergoing a phase of accelerated expansion likely driven for some field (dubbed 'dark energy field') that clusters, if any, only at the largest scales, able to generate a negative pressure large enough to violate the strong energy condition—see [1, 2] and references therein. By far, the conceptually simplest dark energy candidate is the cosmological constant, Λ. Albeit thus far it fits reasonably well all the cosmological data it confronts two serious drawbacks on the theoretical side. On the one hand, its quantum field value results about 123 orders of magnitude larger than observed. On the other hand, it gives rise to the *coincidence problem*, namely: 'why are the vacuum and dust energy densities of precisely the same order today?' (Bear in mind that the energy density of dust red-shifts with expansion as a^{-3}, where a denotes the scale factor of the Robertson–Walker metric.) This is why a number of candidates of varying degree of plausibility have been proposed over the last years with no clear winner in sight—see [3] for a recent review. Here we focus on a dark energy candidate grounded on sound thermodynamic considerations that is receiving growing attention in the literature, namely, the 'holographic dark energy'.

2. Holographic dark energy

We begin by briefly introducing the holography concept after 't Hooft [4] and Susskind [5]. Consider the world as three-dimensional lattice of spin-like degrees of freedom and assume that the distance between every two neighbouring sites is some small length ℓ. Each spin can be in one of two sates. In a region of volume L^3 the number of quantum states will be $N(L^3) = 2^n$, with $n = (L/\ell)^3$ the number of sites in the volume, whence the entropy will be $S \propto (L/\ell)^3 \ln 2$. One would expect that if the energy density does not diverge, the maximum entropy varies as L^3, i.e., $S \sim L^3\Lambda^3$, where $\Lambda \equiv \ell^{-1}$ is to be identified with the ultraviolet cutoff. However, the energy of most states so described would be so big that they will collapse to a black hole larger than L^3. It seems therefore reasonable that in the quantum theory of gravity the maximum entropy should be proportional to the area, not the volume, of the system under consideration. (Recall that the Bekenstein–Hawking entropy is $S_{\rm BH} = A\big/\big(4\ell_{Pl}^2\big)$, where A is the area of the black hole horizon.)

Consider now a system of volume L^3 of energy slightly below that of a black hole of the same size but with entropy larger than that of the black hole. By hurling in a tiny amount of energy a black hole would result but with smaller entropy than the original system thus violating the second law of thermodynamics. As a consequence, Bekenstein suggested that the maximum entropy of the system should be proportional to its area rather than to its volume [6]. In the same vein 't Hooft conjectured that it should be possible to describe all phenomena within a volume by the set of degrees of freedom residing on its boundary. The number of degrees of freedom should not exceed that of a two-dimensional lattice with about one binary degree of freedom per Planck area.

Inspired by these ideas, Cohen *et al* [7] argued that an effective field theory that saturates the inequality $L^3\Lambda^3 \leqslant S_{\rm BH}$ necessarily includes many states with $R_s > L$, where R_s is the Schwarzschild radius of the system under consideration. It seems therefore reasonable to propose a stronger constraint on the infrared cutoff L that excludes all states lying within R_s, namely, $L^3\Lambda^4 \leqslant m_{Pl}^2 L$ (clearly, Λ^4 is the zero-point energy density associated with the short-distance cutoff). So, we may conclude that $L \sim \Lambda^{-2}$ and $S_{\max} \simeq S_{\rm BH}^{3/4}$. By saturating the inequality—which is not compelling at all—and identifying Λ^4 with the holographic dark energy density one has [8]

$$\rho_x = 3c^2 M_p^2\big/L^2 \qquad \big(M_p^2 \equiv (8\pi G)^{-1}\big), \tag{1}$$

where c^2 is a dimensionless constant.

Suggestive as they are, the above ideas provide no clue about how to choose the infrared cutoff in a cosmological context. Different possibilities have been tried with varying degrees of success, namely, the particle horizon [9], the future event horizon [8, 10] and the Hubble horizon [11, 12]. Here we shall adhere to the latter for it looks the most natural one.

3. Interacting dark energy

Our model rests on three main assumptions: (i) the dark energy density is given by equation (1), (ii) $L = H^{-1}$, where $H \equiv \dot{a}/a$ is the Hubble function, and (iii) matter and holographic dark energy do not conserve separately but the latter decays into the former with rate $\Gamma > 0$, i.e.,

$$\dot{\rho}_m + 3H\rho_m = \Gamma\rho_x, \tag{2}$$

$$\dot{\rho}_x + 3H(1+w)\rho_x = -\Gamma\rho_x. \tag{3}$$

Interacting dark energy was first introduced by Wetterich [13], and in a holographic setting by Horvat [14]. In spatially flat universes there is a relation connecting the equation of state parameter of the dark energy to the ratio between the energy densities, $r \equiv \rho_m/\rho_x$, and Γ, namely, $w = -(1+r)\Gamma/(3rH)$, such that any decay of the dark energy into pressureless matter implies a negative w. It also follows that the ratio of the energy densities is a constant, $r_0 = (1-c^2)/c^2$, whatever Γ—see [11] for details.

In the particular case that $\Gamma \propto H$ one has $\rho_m, \rho_x \propto a^{-3m}$ and $a \propto t^n$ with $m = (1+r_0+w)/(1+r_0)$ and $n = 2/(3m)$. Hence, there will be acceleration for $w < -(1+r_0)/3$. In consequence, the interaction is key to simultaneously solve the coincidence problem and have late acceleration. For $\Gamma = 0$ the choice $L = H^{-1}$ does not lead to acceleration. We wish to emphasize that models in which matter and dark energy interact with each other considerably alleviate the coincidence problem [15] and fare remarkably well when measured against observational data [16].

Obviously, prior to the current epoch of accelerated expansion a matter dominated period is required for the standard picture of cosmic structure formation to hold. The usual way to incorporate this is to assume that the ratio r has not been constant but was (and possibly still is) decreasing towards a final value r_0. In the present context, a time dependence of r can only be achieved by allowing the parameter c^2 to vary slowly with time. By 'slowly' we mean that $0 < (c^2)^{\cdot}/c^2 \ll H$. This is not only permissible but reasonable since it is natural to expect that the holographic bounds get fully saturated only in the very long run or even asymptotically. Our approach, however, offers a different way to recover an early matter dominated epoch. Namely, for $\Gamma/H \ll 1$, the dark energy itself behaves as pressureless matter since one has $|w| \ll 1$, even for a constant r. It is straightforward to check that the evolution of the latter is governed by

$$\dot{r} = 3Hr\left[w + \frac{1+r}{r}\frac{\Gamma}{3H}\right]. \tag{4}$$

Likewise, combining Friedmann's equation, $3M_p^2H^2 = \rho_m + \rho_x$, with $\rho_x = 3M_p^2c^2(t)H^2$ we obtain $c^2(t) = \frac{1}{1+r(t)}$. At late times, $r \to r_0$ whence $c^2 \to c_0^2$. In this scenario w depends also on the fractional change of c^2 according to

$$w = -\left(1+\frac{1}{r}\right)\left[\frac{\Gamma}{3H} + \frac{(c^2)^{\cdot}}{3Hc^2}\right]. \tag{5}$$

Since the holographic dark energy must satisfy the dominant energy condition (and therefore it is not compatible with 'phantom energy' [17]), the restriction $w \geqslant -1$ sets constraints on Γ and c^2.

For future convenience we write the deceleration parameter

$$q = \tfrac{1}{2}\Omega_m + \tfrac{1}{2}(1+3w)\Omega_x, \tag{6}$$

where Ω_m and Ω_x stand for the dimensionless density parameters of matter and dark energy, respectively. Up to now we have restricted our attention to spatially flat Friedmann–Lemaitre–Robertson–Walker (FLRW) universes. It proves illustrating to extend the study to FLRW models with curved spatial sections.

3.1. FLRW universes with $k \neq 0$

Aside from the sake of generality other motivations for allowing models with non-flat spatial sections are as follows: (i) inflation drives the k/a^2 ratio close to zero but it cannot set it to zero if $k \neq 0$ initially. (ii) The closeness to perfect flatness depends on the number of e-folds and we can only speculate about the latter. (iii) After inflation the absolute value of the k/a^2 term in Friedmann's equation is bound to steadily increase with respect to the matter density term, thereby the former should not be ignored when studying the late Universe. (iv) Recent observations allow a tiny but not vanishing spatial curvature [2, 18].

For curved spatial sections, equations (4) and (5) generalize to

$$\dot{r} = -3Hr\frac{1}{1-\Omega_x}\left\{\frac{k}{a^2H^2}\left[\frac{1}{r}\frac{\Gamma}{3H}-\frac{1}{3}\right]+\frac{1}{3H}\frac{(c^2)^{\cdot}}{c^2}\right\}, \tag{7}$$

and

$$w = -\frac{1}{1-\Omega_x}\left[\frac{\Gamma}{3H}-\frac{1}{3}\frac{k}{a^2H^2}+\frac{1}{3H}\frac{(c^2)^{\cdot}}{c^2}\right], \tag{8}$$

respectively—see [12] for details. We see that, aside from the evolution of c^2, the evolution of the matter–dark energy ratio r is immediately connected to a non-vanishing spatial curvature which may help to speed the decrease of the former. On the other hand, because $0 < (c^2)^{\cdot}/c^2 \ll H$ by assumption and $|k/(aH)^2|_0 \ll 1$ by observation it follows that $|\dot{r}/r|_0 \ll H_0$, i.e., the coincidence problem gets greatly alleviated (bear in mind that in the conventional ΛCDM scenario $|\dot{r}/r|_0 = 3H_0$).

Likewise, the curvature term modifies the equation of state parameter w. Depending on the whether the Universe is spatially open or closed the negative character of w will be accentuated or softened. A detailed analysis of the impact of the curvature term on this and related issues as the transition from deceleration to acceleration can be found in [12].

4. Transition to a new decelerated era?

It has been speculated that the present phase of accelerated expansion is just transitory and that the Universe will eventually revert to a fresh decelerated era. This can be achieved by taking as dark energy a scalar field whose energy density obeys a suitable ansatz. As a result the equation of state parameter w evolves from values above but close to -1 to much less negative values thereby the deceleration parameter increases to positive values [19]. Thus, the troublesome event horizon that afflicts superstring theories disappears altogether. Here we shall argue that our holographic interacting model—which was devised to provide a transition from deceleration to acceleration and alleviate the coincidence problem—is in principle compatible with such a transition.

For the sake of simplicity we set $k = 0$. Inspection of equation (5) reveals that w can become larger than $-1/3$ (which by equation (6) means deceleration) either by allowing any of the two terms in the square parenthesis, or both, to reach sufficiently small values or just keeping the first term nearly constant and allowing the second one to become negative enough. Clearly, all these possibilities look a bit contrived, especially, the latter one as—contrary to intuition—in such a case, the saturation parameter does not increase but decreases. However, we should no wonder at this as the proposal of coming back to a decelerated phase for the sole purpose of getting rid of the event horizon appears rather artificial, especially because nothing in the observational data hints at that. Nonetheless, we should keep an open mind since this possibility cannot be dismissed offhand. At any rate, we wish to emphasize that those holographic dark energy models that identify the infrared cutoff L with the event horizon are unable to account for such a transition.

In all, the holographic dark energy provides a simple and elegant thermodynamic-based explanation, within Einstein relativity, for the present era of cosmic accelerated expansion. Moreover, it substantially alleviates the coincidence problem provided that matter and dark energy do not conserve separately. Finally, the model can, in principle, accommodate a later transition to a new decelerated phase. Present and coming observational data should constrain the basic parameters of the model, i.e., Γ and c^2.

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